

AD-A095 140

CALIFORNIA UNIV BERKELEY OPERATIONS RESEARCH CENTER F/G 12/1
AVAILABILITY OF SERIES SYSTEMS WITH COMPONENTS SUBJECT TO VARIO--ETC(U)
JUN 80 Z S KHALIL AFOSR-77-3179
ORC-80-12 NL

UNCLASSIFIED

1 of 1
AD-A095140



END

DATE

FILED

3-81

DTIC

ORC 80-12
JUNE 1980

AVAILABILITY OF SERIES SYSTEMS WITH COMPONENTS
SUBJECT TO VARIOUS SHUT-OFF RULES

by
ZOHEL S. KHALIL

(12)

LEVEL II

AD A095140

OPERATIONS
RESEARCH
CENTER

DTIC
ELECTE
FEB 18 1981
S E

UNIVERSITY OF CALIFORNIA • BERKELEY

81 2 13 050

AVAILABILITY OF SERIES SYSTEMS WITH COMPONENTS

SUBJECT TO VARIOUS SHUT-OFF RULES[†]

by

Zohel S. Khalil^{††}
Department of Mathematics
Concordia University
Montreal, Canada

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

JUNE 1980

ORC 80-12

[†]This research has been partially supported by the Air Force Office of Scientific Research (AFSC), USAF, under Grant AFOSR-77-3179 with the University of California and the Natural Sciences and Engineering Research Council, Grant No. A9095, Canada.

^{††}Visiting Research Engineer, Operations Research Center, University of California, Berkeley.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ORC-80-12 ✓	2. GOVT ACCESSION NO. AD-A095 140	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AVAILABILITY OF SERIES SYSTEMS WITH COMPONENTS SUBJECT TO VARIOUS SHUT-OFF RULES		5. TYPE OF REPORT & PERIOD COVERED Research Report
6. AUTHOR(s) Zohel S. Khalil		7. PERFORMING ORG. REPORT NUMBER
8. CONTRACT OR GRANT NUMBER(s) AFOSR-77-3179		9. PERFORMING ORGANIZATION NAME AND ADDRESS Operations Research Center ✓ University of California Berkeley, California 94720
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2304/A5 DAS		11. CONTROLLING OFFICE NAME AND ADDRESS United States Air Force Air Force Office of Scientific Research Bolling AFB, D.C. 20332
12. REPORT DATE June 1980		13. NUMBER OF PAGES 20
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Reliability Series System Failures Availability Shut-off Rules		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (SEE ABSTRACT)		

DD FORM 1473
1 JAN 73

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ACKNOWLEDGEMENT

I am grateful to Professor Richard Barlow for his constant support and encouragement and for his valuable suggestions and criticism.

C-

ABSTRACT

In this report we consider different shut-off rules for series systems performance. We calculate limiting system availability under various shut-off rules. In particular, availability results for 2 and 3 unit systems with all failure and repair distributions exponential are extended to systems of arbitrary size.

- 1 -

AVAILABILITY OF SERIES SYSTEMS WITH COMPONENTS
SUBJECT TO VARIOUS SHUT-OFF RULES

by

Zohel S. Khalil

0. INTRODUCTION

Consider a series system of n components. System failure in such systems coincides with component failure. However, in many systems components can still be functioning after system failure occurs, or some components on failure will shut off others. This means that some shut-off rules can be established for system performance, such as failure of first component shuts off the second and next components or failure of second component shuts off third and higher components but not the first and so on. For example, in a 3-unit system, unit 1 on failure shuts off both unit 2 and 3, unit 2 on failure shuts off only unit 3 but not 1 and units 1 and 2 are not affected when failure of unit 3 occurs.

In this report we study some of these shut-off rules and calculate system availability under these rules.

1. PRELIMINARIES: AVAILABILITY THEORY

An important measure of system performance is system availability $A(t)$ which is defined as the probability that the system is functioning at time t . Let

$$X(t) = \begin{cases} 1 & \text{if system is up at time } t \\ 0 & \text{otherwise.} \end{cases}$$

Then $A(t)$ is given by:

$$A(t) = \Pr\{X(t) = 1\} = EX(t).$$

Limiting availability or just availability is defined by $A = \lim_{t \rightarrow \infty} A(t)$ when this limit exists.

Consider a system consisting of one unit and suppose at $t = 0$, the system is put to work and will be up for a time U_1 and then failure occurs. The system is then down for a time D_1 until repair or replacement is done. We assume that repair restores all system characteristics, i.e., the system when it starts functioning again is like new. The second uptime is U_2 and downtime is D_2 etc. We assume that the sequence $U_i + D_i$, $i = 1, 2, \dots$ is mutually stochastically independent. Let $F(t)$ be the common distribution of U_i and $H(t)$ be the common distribution of $U_i + D_i$, $i = 1, 2, \dots$. The sequence $U_i + D_i$ forms a renewal process with renewal function

$$M_H(t) = \sum_{k=1}^{\infty} H^{(k)}(t)$$

where $H^{(k)}(t)$ is the k -fold convolution of $H(t)$.

We summarize some well known results [2].

Lemma 1:

System availability at time t is given by

$$A(t) = \bar{F}(t) + \int_0^t \bar{F}(t-u) dM_H(u) .$$

Proof:

$$A(t) = P(\text{system is up at time } t)$$

$$= \bar{F}(t) + \sum_{k=1}^{\infty} \int_0^t \bar{F}(t-u) dH^{(k)}(u)$$

$$= \bar{F}(t) + \int_0^t \bar{F}(t-u) d \sum_{k=1}^{\infty} H^k(u)$$

$$= \bar{F}(t) + \int_0^t \bar{F}(t-u) dM_H(u) .$$

Lemma 2:

Limiting system availability is given by:

$$A = \lim_{t \rightarrow \infty} A(t) = \frac{E(U)}{E(U) + E(D)} .$$

Proof:

This is a direct consequence of the Key Renewal Theorem [2]. Note that limiting availability as given by the last equation depends only on the system's mean time to failure and mean time to repair.

This result holds for a component or a system treated as a single unit [2].

2. AVAILABILITY OF SERIES SYSTEMS

Consider now a system with n components in series. System failure in this case coincides with component failure. In the sequel, we will investigate 3 different models of series system performance. The first model discussed in full detail by Barlow and Proschan [1], [2] will be called:

Model A:

When a failed component is replaced or repaired, all other components are in a state of suspended animation, i.e., cannot fail and do not age. In other words, failure of any one component shuts off all others. We denote such a system by a double arrow connection (see Figure 1).

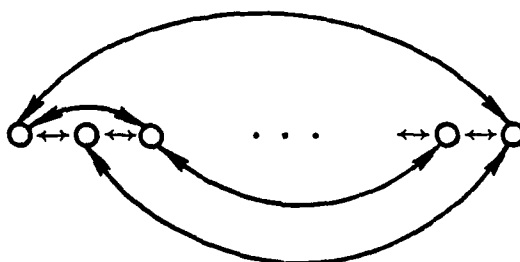


FIGURE 1

The limiting availability in this case is:

$$A = \left(1 + \sum_{i=1}^n \frac{v_i}{\mu_i} \right)^{-1}$$

where μ_i and v_i are mean time to failure and repair, respectively.

Let $N_i(t)$ be the number of system failures due to component i in $[0, t]$, then

$$\lim_{t \rightarrow \infty} \frac{N_i(t)}{t} = \lim_{t \rightarrow \infty} \frac{E(N_i(t))}{t} = A/\mu_i.$$

If $N(t)$ is the total number of system failure in $[0, t]$, then

$$N(t) = \sum_{i=1}^n N_i(t)$$

and

$$\lim_{t \rightarrow \infty} \frac{N(t)}{t} = A \sum_{i=1}^n \frac{1}{\mu_i}.$$

Let U_i be the successive system uptimes, then

$$\mu = \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^{N(t)} U_i}{N(t)} = \left(\sum_{i=1}^n \frac{1}{\mu_i} \right)^{-1}.$$

Let D_i be the successive system downtimes, then

$$\nu = \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^{N(t)} D_i}{N(t)} = \mu \sum_{i=1}^n \frac{\nu_i}{\mu_i} = \mu \sum_{i=1}^n \rho_i$$

where $\rho_i = \frac{\nu_i}{\mu_i}$.

It follows that $A = \left(1 + \sum_{i=1}^n \rho_i \right)^{-1} = \mu / \left(\mu + \mu \sum_{i=1}^n \frac{\nu_i}{\mu_i} \right) = \mu / (\mu + \nu)$.

This is the analog of the one system availability defined before.

Consider a series system with shut-off rule as in Model A.

Furthermore, let components have exponential failure distribution

$$F_1(t) = 1 - \exp[-\lambda_1 t] \text{ and repair distributions } G_1(t) = 1 - \exp[-\theta_1 t] .$$

It is easy to prove the following:

Lemma 3:

The system downtime distribution is a mixture of G_1 with weights

$$\lambda_i / \sum_{j=1}^n \lambda_j \quad i = 1, 2, \dots, \text{ i.e.,}$$

$$G(t) = \sum_{i=1}^n \lambda_i G_1 / \sum_{i=1}^n \lambda_i = 1 - \sum_{i=1}^n \lambda_i \exp[-\theta_i t] / \sum_{i=1}^n \lambda_i .$$

The Laplace transform of the survival function $\bar{G}(t)$ in this case is given by:

$$\bar{G}^*(s) = \int_0^{\infty} e^{-st} \bar{G}_n(t) dt = \frac{1}{\sum_{i=1}^n \lambda_i} \sum_{i=1}^n \frac{\lambda_i}{\theta_i + s} .$$

Model B:

Consider a 2-unit system in series with the following shut-off rule: unit one upon failure shuts off two but not vice versa. We symbolize this case by one arrow connection $o \rightarrow o$. Barlow and Hudes [4] prove the following theorem.

Theorem 1:

Let F_1 and G_1 be exponential; F_2, G_2 arbitrary distributions.

Then

$$A_{0 \rightarrow 0} = (1 + \rho_1)^{-1} \left\{ 1 + \frac{v_2}{\mu_2} \left[\frac{1}{1 + \rho_1} + \frac{1}{1 + \rho_1} \cdot \frac{1}{v_2} \bar{G}_2^* \left(\frac{1}{\mu_1} + \frac{1}{v_1} \right) \right] \right\}^{-1}.$$

We observe that this depends on F_2 only through its mean μ_2 but the dependence on G_2 is more involved namely through its mean v_2 and the Laplace transform of its survival function at $\frac{1}{\mu_1} + \frac{1}{v_1}$.

3. COMBINATION OF SHUT-OFF RULES

Consider a series system of n components with the following shut-off rule: component 1 shuts off all other $n-1$ components but not vice versa and the other $n-1$ components follow the shut-off rule of Model A. Symbolically as given by Figure 2,

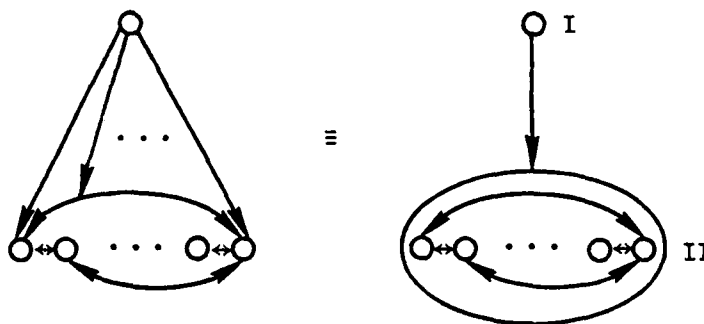


FIGURE 2

It follows from this last diagram that subsystems I and II are interrelated by the shut-off rule of Model B. If F_I and G_I are exponential then the conditions of the last theorem are satisfied and hence we have the following:

Theorem 2:

Limiting system availability is given by:

$$A = (1 + \rho_1)^{-1} \left[1 + \frac{v_{II}}{\mu_{II}} \left(\frac{1}{1 + \rho_1} + \frac{1}{1 + \rho_1} \cdot \frac{1}{v_{II}} \cdot \frac{1}{\sum_{i=2}^n \lambda_i} \sum_{i=2}^n \frac{\lambda_i}{\lambda_1 + \theta_1 + \theta_i} \right) \right]^{-1}$$

where we have assumed that failure and repair distributions are exponential in Subsystem II and hence Lemma 3 gives its repair distribution, where

$$\mu_{II}^{-1} = \sum_{i=2}^n \lambda_i, \text{ and } v_{II} = \left(\frac{1}{\sum_{i=2}^n \lambda_i} \right) \sum_{i=2}^n \frac{\lambda_i}{\theta_i}.$$

Proof:

It is easy to verify the Markov conditions used in the proof of Theorem 1 in Barlow and Hudes (1979).

Model C:

In this model nonfailed components continue to operate regardless of the number of failed components. More general coherent structures were treated by Ross [3]. System availability in the series case is given by:

$$A = \prod_{i=1}^n \frac{1}{1 + \frac{v_i}{\mu_i}}.$$

The limiting average of uptimes is

$$\mu = \left(\sum_{i=1}^n \frac{1}{\mu_i} \right)^{-1}.$$

The limiting average of downtimes is

$$v = \frac{1 - A}{A} \mu.$$

Denote $\rho_c = \frac{v}{\mu} = \frac{1}{A} - 1 = \prod_{i=1}^n (1 + \rho_i) - 1$. Consider now the following system: one unit shuts off the other $n-1$ and vice versa but these last behave as in Model C or symbolically as in Figure 3.

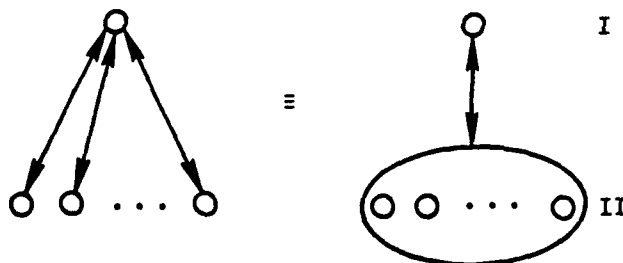


FIGURE 3

We observe that the first unit 1 and the subsystem II of $n-1$ units are related by the shut-off rule of Model A. Hence

Theorem 3:

System availability is given by:

$$A = \frac{1}{1 + \rho_1 + \rho_{II}} \quad (1)$$

where ρ_{II} is the coefficient of subsystem II having shut-off rule of Model C. Hence:

$$\rho_{II} = \prod_{i=2}^n (1 + \rho_i) - 1$$

and $A = \left[\rho_1 + \prod_{i=2}^n (1 + \rho_i) \right]^{-1}.$

Example 1:

Consider the system shown in Figure 4. We have

$$\rho_{II} = (1 + \rho_2)(1 + \rho_3) - 1 = \rho_2 + \rho_3 + \rho_2\rho_3$$

and hence

$$A = (1 + \rho_1 + \rho_2 + \rho_3 + \rho_2\rho_3)^{-1}.$$

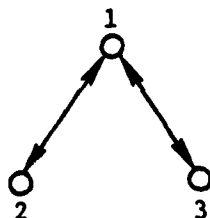


FIGURE 4

Remark:

Subsystem II as in Theorem 3 can have any configuration of shut-off rules as long as between I and II, Model A rule holds.

Example 2:

Consider the system shown in Figure 5.

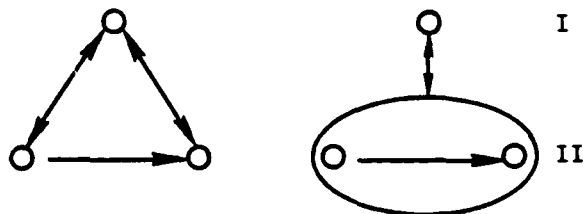


FIGURE 5

By (1), system availability is

$$A = \frac{1}{1 + \rho_1 + \rho_{0 \rightarrow 0}}$$

but from Model B, see [5] $\rho_{0 \rightarrow 0} = \rho_2 + \rho_3(1 + \rho_2) \frac{\theta_2 + \theta_3}{\lambda_2 + \theta_2 + \theta_3}$. Hence

$$A = \left[1 + \rho_1 + \rho_2 + \rho_3(1 + \rho_2) \frac{\theta_2 + \theta_3}{\lambda_2 + \theta_2 + \theta_3} \right]^{-1}.$$

Example 3:

Consider the following system (Figure 6):

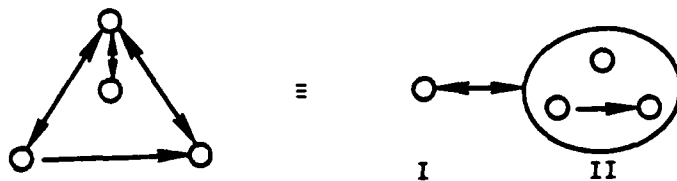


FIGURE 6

We have:

$$A = (1 + \rho_I + \rho_{II})^{-1}$$

$$\rho_{II} = (1 + \rho_2)(1 + \rho_3) \left[1 + \rho_4 \frac{\theta_3 + \theta_4}{\lambda_3 + \theta_3 + \theta_4} \right] \quad (\text{see [5]}).$$

Substituting back in the expression for A , we get the explicit availability.

Next, we investigate systems of n units in which one unit has failure independent of the other $n-1$ units (Model C) but these last can have different shut-off rules. Symbolically (see Figure 7):

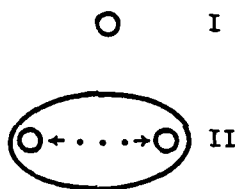


FIGURE 7

It is easy to show that system availability in this case is given by:

$$A = \{(1 + \rho_I)(1 + \rho_{II})\}^{-1}$$

where we used Model C, ρ_I and ρ_{II} are coefficients of component I and subsystem II respectively.

Example 4:

We find the availability of system of Figure 8

$$A = \{(1 + \rho_1)[1 + \rho_{O \rightarrow O}]\}^{-1}$$

but

$$\rho_{0 \rightarrow 0} = (1 + \rho_2) \left[1 + \rho_3 \frac{\theta_2 + \theta_3}{\lambda_2 + \theta_2 + \theta_3} \right] - 1 .$$

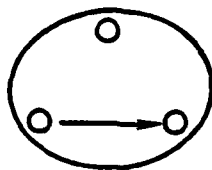


FIGURE 8

Substituting back, we get the explicit expression for A .

Example 5:

Consider the system of Figure 9. We have:

$$\begin{aligned} A &= \frac{1}{(1 + \rho_I)(1 + \rho_{II})} \\ &= \left\{ (1 + \rho_1) \left[1 + \sum_2^4 \rho_i \right] \right\}^{-1} \end{aligned}$$

by Model A.

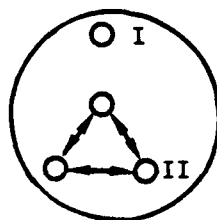


FIGURE 9

Consider again the 2 component system as in Model B. Barlow and Hudes [4] proved the following result:

Theorem 4:

Let F_1 , F_2 and G_2 be exponential with means μ_1 , μ_2 and ν_2 respectively, G_1 arbitrary, then system availability $A(G_1)$ is given by:

$$A(G_1) = \left\{ (1 + \rho_1) \left[1 + \frac{\rho_2}{1 + \frac{1}{\mu_1} \bar{G}_1^* \left(\frac{1}{\nu_2} \right)} \right] \right\}^{-1}.$$

Now suppose we have n units in series such that $n-1$ follow shut-off rule of Model A, but all with the first unit follow shut-off rule of Model B.

Symbolically:

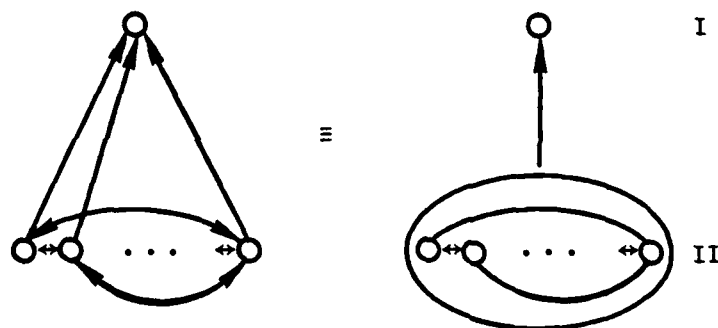


FIGURE 10

Furthermore, suppose all failure and repair distributions are exponential with respective failure and repair rates. By Model A results, subsystem II will have then an exponential failure distribution with rate $\lambda_2 + \lambda_3 + \dots + \lambda_n$ and by Lemma 3, a mixture of repair distributions of units $2, 3, \dots, n$ with weights $\lambda_i / \sum_{i=2}^n \lambda_i$ $i = 2, 3, \dots, n$. The Laplace transform of the survival function as before is

$$\bar{G}_{II}^*(s) = \frac{1}{\sum_{i=2}^n \lambda_i} \sum_{i=2}^n \frac{\lambda_i}{\theta_i + s}$$

and the conditions of Theorem 4 are satisfied, hence

Theorem 5:

System availability is given by:

$$A = \left\{ \left(1 + \sum_{i=2}^n \rho_i \right) \left[1 + \rho_1 \left(1 + \sum_{i=2}^n \frac{\lambda_i}{\theta_i + \theta_1} \right)^{-1} \right] \right\}^{-1}.$$

REFERENCES

- [1] Barlow, R. E. and F. Proschan, "Availability Theory for Multicomponent Systems," in MULTIVARIATE ANALYSIS - III, Academic Press, New York, pp. 319-335 (1973).
- [2] Barlow, R. E. and F. Proschan, STATISTICAL THEORY OF RELIABILITY AND LIFE TESTING: PROBABILITY MODELS, Holt, Rinehart and Winston, 1975.
- [3] Ross, S. M., "On the Calculation of Asymptotic System Reliability Characteristics," in RELIABILITY AND FAULT TREE ANALYSIS, R. E. Barlow, J. B. Fussell and N. D. Singpurwalla (eds.), SIAM, pp. 331-350 (1975).
- [4] Barlow, R. E. and E. S. Hudes, "Asymptotic Measures of System Performance under Alternative Rules, I & II, ORC 79-6 and ORC 79-7, Operations Research Center, University of California, Berkeley (1979).
- [5] Hudes, E. S., "Availability Theory for Systems Whose Components are Subjected to Various Shut-Off Rules," Ph.D. dissertation, Department of Statistics, University of California, Berkeley, California (1979).

